CS310 & NEEP271 **SECOND MIDTERM EXAM** Fall 1999
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There are 25 Questions on this Exam.
There is one and only one answer for each question.
No Books, Notes, Notecards, or Calculators are allowed.

1. The result of the Maple commands

```
A:= array( [ [ 1, 0 ], [ a, 1] ] ) ;
B:= array( [ [ 1, 0 ], [ b, 1] ] ) ;
C:= evalm( A &* B ) ;
```

is

a.) $\begin{pmatrix} 1 & 0 \\ ab & 1 \end{pmatrix}$ b.) $\begin{pmatrix} 1 & 0 \\ a+b & 1 \end{pmatrix}$

c.) $\begin{pmatrix} 1 & ab \\ 0 & 1 \end{pmatrix}$ d.) $\begin{pmatrix} 1 & a+b \\ 0 & 1 \end{pmatrix}$

Answer: b

2. We wish to get the solution of the ordinary differential equation

$$\frac{d\,q(t)}{dt} = q(t)^2 - 1$$

with initial conditions $q(0) = 2$. Which of the following Maple commands, and which order, will give the solution.

```
1. de:= dsolve( {de, D(q)(0) = 2 }, q(t) );
2. de:= dsolve( {de, q(0) = 2 }, q(t) );
3. de:= diff( de , t ) ;
4. de:= q(t)^2 - 1 ;
5. de:= diff( q(t), t ) = q(t)^2 - 1;
```

a.) 5, 1, 3 b.) 4, 2 c.) 5, 2
d.) 4, 1 e.) 5, 1

Answer: c

3. If `a` is a Matlab row vector with 4 components and `b` is a Matlab row vector with 5 components, which one of the following conclusions is correct concerning the Matlab statement
`w = [a b].`

   a.) This is an invalid Matlab statement, `a` and `b` must have the same length.

   b.) This is a valid Matlab statement, `w` is a vector with 9 components.

   c.) This is a valid Matlab statement, `w` is a scalar equal to

   ```
   a(1)*b(1) + a(2)*b(2) + a(3)*b(3) + a(4)*b(4).
   ```

   Answer: b

4. In the Matlab statement

   ```
   [ t, y ] = ode45( 'motion' , tstart, tstop, ystart ) ;
   ```

   a.) `motion` refers to the array where the solution is stored.

   b.) `motion` refers to the array where the derivative of the solution is stored.

   c.) `motion` refers to the function that computes the derivative of $y(t)$.

   d.) `motion` refers to the function that computes $y(t)$.

   Answer: c

5. Which Matlab statements will create a 5×5 matrix with ones across the last row and zeros elsewhere?

   a.)
   ```
   for k = 1 to 5
      for j = 1 to 5
         A(k,j) = 1
      end
   end
   ```

   b.)
   ```
   B = zeros(5,4)
   A = ones(5,1)
   C = [ B A]
   ```

   c.)
   ```
   for k = 1:5
      A(k,5) = 1
   end
   ```

   d.)
   ```
   B = zeros(5,4)
   A = ones(5,1)
   C = A + B
   ```

   e.)
   ```
   B = zeros(4,5)
   A = ones(1,5)
   C = [ B ; A]
   ```

   Answer: e

6. Which Matlab statement is correct for obtaining a vector where the elements are formed using

$$y = x^2 \cos(x) + \sin(x)$$

where `x` has been defined as `x = 0:.01:2*pi` ?

a.) `y = x.^2.*cos(x) + sin(x)`
b.) `y = x.*x*cos(x) + sin(x)`
c.) `y = x.^2*cos(x).+ sin(x)`
d.) `y = x^2.*cos(x.).+ sin(x.)`
e.) None of these.

Answer: a

7. If the arrays `U` and `V` contain values of functions $U(t)$ and $V(t)$ at the values of t in the array `t,` what does this Matlab code do?

```
plot(t, U, t, V, '--' ) ;
```

a.) prints a graph of $U(t) - V(t)$
b.) prints two graphs, one for $U(t)$ and one for $V(t)$
c.) prints a graph of $U(t)$ and $V(t)$ on the same set of axes
d.) prints a graph of $U(t)$ and a graph of $V(t)$ and a dotted line connecting the two.

Answer: c

8. Consider the following Matlab function

```
function output = midterm( z )
   x = z(1) ;
   y = z(2) ;
   w1 = x^2 + y^2 ;
   w2 = x ;
   output = [ w1 ; w2 ]
```

When `z = [ 2 -1 ]` what is the value of `output?`

a.) $\begin{pmatrix} 5 & -1 \end{pmatrix}$ b.) $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ c.) $\begin{pmatrix} 5 & 2 \end{pmatrix}$ d.) $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$

Answer: d

9. What is the result of running the following Matlab code?

```
x=3;
if x==3
   x=4;
   if x==4
      x=5;
   else
      x=6;
   end
end
x
```

   a.) 3   b.) 4   c.) 5   d.) 6

   Answer: c

10. If using Matlab to solve the linear system $Ax = b$ by Gaussian elimination, the correct command is:
    a.) ans $= b\backslash A$
    b.) ans $= A\backslash b$
    c.) ans $= b/A$
    d.) ans $= A^{-1}b$
    e.) You can't do Gaussian elimination on Matlab.

    Answer: b

11. Find the correct Matlab statement for constructing the following row vector:

    x = $[0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \quad 2.5 \quad 3.0 \quad 3.5 \quad 4.0]$

    a.) `x =` 0 : 4 : 0.5   b.) `x =` 0 : 0.5 : 4.0
    c.) `x =` 0 : 0.4 : 5.0   d.) `x =` 0 : 0.4 : 5.0

    Answer: b

12. Which one of the following statements is correct about these Matlab statements?

```
a = 2:4
for i = a
   x(i) = i^2 ;
end
```

    a.) This is not correct, the for loop must be for `i` in an interval, not a vector.
    b.) These statements define the vector `x = [4, 9, 16]`.
    c.) These statements define the vector `x = [0, 4, 9, 16]`.
    d.) These statements define the vector `x = [0, 4, 16]`.

    Answer: c

13. Euler's method is a second-order accurate method for solving ordinary differential equations.

    a.) True   b.) False

    Answer: b

14. The formula for Euler's method for solving an ordinary differential equation is:

    a.) $y_{k+1} = y_k + hf(t_{k+1}, y_k)$ b.) $y_{k+1} = y_k - hf(t_k, y_k)$

    c.) $y_{k+1} = y_k + hf(t_k, y_k)$ d.) $y_{k+1} = y_k - hf(t_{k+1}, y_{k+1})$

    e.) None of the above

    Answer: c

15. The standard form of the differential equation

$$q''' = q'' + (q')^2 - tq$$

is

a.) $y'' = \int_0^t y''dt + \int_0^t (y')^2 dt - t\int_0^t ydt$ b.) $y_3' = y_3 + (y_2)^2 - ty_1$

c.) $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}' = \begin{pmatrix} y_3 + (y_2)^2 - ty_1 \\ y_3 \\ y_2 \end{pmatrix}$ d.) $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}' = \begin{pmatrix} y_2 \\ y_3 \\ y_3 + (y_2)^2 - ty_1 \end{pmatrix}$

e.) $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}' = \begin{pmatrix} y_2 \\ y_3 \\ y_4 \\ y_3 + (y_2)^2 - ty_1 \end{pmatrix}$

Answer: d

16. The ordinary differential equation

$$\frac{d\,f(t)}{dt} = b(r - f(t) + (1-r)f(t)^2)$$

models the fraction $f(t)$ of nonconformists in a population. The parameter b is the birth rate and r is the fraction of nonconformist children from conformist parents. What are the steady states for this ordinary differential equation?

a.) $f(t) = 1$ is the only steady state.
b.) $f(t) = r/(1-r)$ is the only steady state.
c.) $f(t) = 1$ and $f(t) = r$ are the only steady states.
d.) $f(t) = 1$ and $f(t) = r/(1-r)$ are the only steady states.
e.) This equation has no steady state solutions.

Answer: d

17. If Euler's method is used to solve the ordinary differential equation $y' = t - y$ from $t = 1$ where $y(1) = 1$ with step size $h = 1/10$ the first three values of y_k are:

a.) $y_0 = 1$	$y_1 = 1$	$y_2 = 1.01$
b.) $y_0 = 1$	$y_1 = 1$	$y_2 = 1.$
c.) $y_0 = 1$	$y_1 = 1.01$	$y_2 = 1.029$
d.) $y_0 = 1$	$y_1 = 1.2$	$y_2 = 1.43$
e.) $y_0 = 1.2$	$y_1 = 1.43$	$y_2 = 1.693$

Answer: a

18. The formula for the improved Euler's method for solving the ordinary differential equation $y' = f(t, y)$ is:

   a.) $\tilde{y}_{k+1} = y_k - hf(t_k, y_k)$ $\quad y_{k+1} = y_k - \frac{1}{2}h\left(f(t_k, y_k) + f(t_{k+1}, \tilde{y}_{k+1})\right)$

   b.) $\tilde{y}_{k+1} = y_k + hf(t_k, y_k)$ $\quad y_{k+1} = y_k + \frac{1}{2}h\left(f(t_k, y_k) + f(t_k, \tilde{y}_{k+1})\right)$

   c.) $\tilde{y}_{k+1} = y_k + hf(t_k, y_k)$ $\quad y_{k+1} = y_k + \frac{1}{2}h\left(f(t_k, y_k) + f(t_{k+1}, \tilde{y}_{k+1})\right)$

   d.) $\tilde{y}_{k+1} = y_k + hf(t_k, y_k)$ $\quad y_{k+1} = y_k + h\left(f(t_k, y_k) + f(t_{k+1}, \tilde{y}_{k+1})\right)$

   e.) $\tilde{y}_{k+1} = y_k - hf(t_k, y_k)$ $\quad y_{k+1} = y_k - h\left(f(t_k, y_k) + f(t_{k+1}, \tilde{y}_{k+1})\right)$

   Answer: c

19. Consider the following portion of a spreadsheet program for solving the ordinary differential equation $q'(t) = q(t) + 2t$ starting with $q(0) = 2$. The method is the improved Euler's method using a step size of $1/10$.

	A	B	C	D	E	F
1	t	q(t)	q prime	t next	q tilde	q til. pr.
2	0.000	2.000	2.000	0.1000	2.200	2.400
3						

   The two entries in columns A and B of row 3 should be

   a.) 0.050 2.120 b.) 0.100 2.220

   c.) 0.100 2.200 d.) 0.100 2.110

   e.) 0.100 2.240

   Answer: b

20. Given N distinct data points (x_1, y_1), $(x_2, y_2), ..., (x_N, y_N)$, which of the following statements is true?
   a.) There is a unique polynomial of degree $N-1$ that interpolates these points.
   b.) There is a unique polynomial of degree N that interpolates these points.
   c.) There is a unique polynomial of degree $N+1$ that interpolates these points.
   d.) Unless the data is smooth enough there need not be any polynomial that interpolates the data points.
   e.) None of the above

   Answer: a

21. Consider the following divided difference table.

x_k	y_k			
-2	4			
		-3		
-1	1		A	
		1		B
0	2		-2	
		-5		
2	-8			

   The correct values for A and B in the table are:
   a.) $A = 1$ and $B = 2$
   b.) $A = 1$ and $B = 3/4$
   c.) $A = 2$ and $B = 1$
   d.) $A = 2$ and $B = -1$
   e.) None of the above

   Answer: d

22. Consider the following divided difference table.

x_k	y_k			
-1.0	3.0			
		0.5		
-0.5	3.25		-2.0	
		-2.5		2.0
0.5	0.75		2.0	
		0.5		
1.0	1.0			

The polynomial that interpolates the data is :

a.) $-1.0 + (x+3)\Big[\ -0.5 + (x-3.25)\Big[\ 0.5\ \ + (x-0.25)\big[\ 0.5\ \big]\Big]\Big]$

b.) $3.0 + (x+1)\Big[\ 0.5\ \ + (x+0.5)\Big[\ -2.0 + (x-0.5)\big[\ 2.0\ \big]\Big]\Big]$

c.) $3.0 + (x+0.5)\Big[\ 0.5\ \ + (x-0.5)\Big[\ -2.0 + (x-1.0)\big[\ 2.0\ \big]\Big]\Big]$

d.) $3.0 + (x-1)\Big[\ 0.5\ \ + (x-0.5)\Big[\ -2.0 + (x+0.5)\big[\ 2.0\ \big]\Big]\Big]$

e.) None of the above

Answer: b

23. The value of the polynomial

$$1.0 + (x-1)\Big[\ 0.5\ \ + (x-0.5)\Big[\ 2.0 + (x+0.5)\big[\ 2.0\ \big]\Big]\Big]$$

at $x = 0$ is

a.) -1 b.) 1 c.) 2 d.) 3 e.) 0.5

Answer: c

24. Consider the five conditions.
    1. $S(x)$ is continuous.
    2. $S'(x)$ is continuous.
    3. $S''(x)$ is continuous.
    4. $S'''(x)$ is continuous.
    5. Between adjacent knots $S(x)$ is a cubic polynomial in x.

    The function $S(x)$ is *defined* to be a cubic spline if it satisfies

    a.) conditions 1, and 5
    b.) conditions 1, 2, and 5
    c.) conditions 1, 2, 3, and 4
    d.) conditions 1, 3, and 4
    e.) conditions 1, 2, 3, and 5

    Answer: e

25. Is the following a cubic spline?

$$S(x) = \begin{cases} 1 + x + x^2 - x^3 & \text{for } 0 \leq x \leq 1 \\ 2 - 2(x-1)^2 & \text{for } 1 \leq x \leq 2 \end{cases}$$

    a.) Yes
    b.) No

    Answer: a