CS310 **SAMPLE QUESTIONS FOR MIDTERM EXAM** Fall 1999

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There is one and only one answer for each question.

1. $(1357)_8$ means
   a.) $1(8) + 3(8)^2 + 5(8)^3 + 7(8)^4$
   b.) $1(8)^3 + 3(8)^2 + 5(8) + 7$
   c.) $1(1) + 3(8) + 5(8)^2 + 7(8)^3$
   d.) $1(8)^4 + 3(8)^3 + 5(8)^2 + 7(8)^1$
   e.) $8^1 + 8^3 + 8^5 + 8^7$

2. The term "unit roundoff" for a computer refers to
   a.) The smallest number that can be represented on the computer
   b.) The largest number that can be represented on the computer
   c.) The largest number such that if one more digit is added, the computer will round off the number
   d.) The smallest number which when added to 1.0 gives 1.0
   e.) The largest number which when added to 1.0 gives 1.0

3. In a computer using 7 decimal digits, how would the result of the following assignment be stored?

```
x = -4356.2398423
```

a.)

+	4356239	+04

b.)

−	4356239	+05

c.)

−	4356240	+05

d.)

+	4356239	−04

e.)

−	4356240	+04

4. The function $f(x)$ is known to be continuous and differentiable and has at least one root in the interval $[-2 : 1]$. Suppose you wish to use Newton's method to find a root in that interval. Which of the following should be done?
   a.) Use -2 as the initial approximation because then Newton 's method works from the left to the right.
   b.) Use the midpoint -0.5 as the initial approximation since it is closer to the root and will make the method converge faster.
   c.) Since you need two points to start Newton's method, use the two endpoints.
   d.) You may have to try several starting points before finding one that converges to a root in the interval.
   e.) If Newton's method converges to a root outside $[-2 : 1]$ for your starting point in $[-2 : 1]$, it means Newton's method will not work for this function.

5. Consider the method of successive substitution for finding the solution of $x = 2 - e^x$ in the interval $[0 : 1\,]$ starting with $x_0 = 0$. Which of the following statements is true?
   a.) Because $|e^x| < 1$ near the solution, the method converges.
   b.) Because $|e^x| > 1$ near the solution, the method converges.
   c.) Because $|e^x| < 1$ near the solution, the method diverges.
   d.) Because $|e^x| > 1$ near the solution, the method diverges.
   e.) With the information given, we can not say if the method will converge or diverge.

Problems 6 and 7 relate to the following portion of a spreadsheet program for finding the root of the polynomial equation $x^3 + 4x^2 - 10 = 0$ by the secant method.

	A	B	C	D	E
0	x0	x1	f(x0)	f(x1)	slope
1	1.0000000	2.0000000	-5.0000000	14.0000000	19.0000000
2	2.0000000	1.2631579	14.0000000	-1.6022744	21.1745152
3	1.2631579	1.3388278	-1.6022744	-0.4303647	15.4871217
4	1.3388278	1.3666164	-0.4303647	0.0229094	16.3115414
5	1.3666164	1.3652119	0.0229094	-0.0002991	16.5244781

6. Of the values in the table, what is the best estimate for the root of the equation?

a.) 1.3666164

b.) 1.3652119

c.) 0.0229094

d.) -0.0002991

e.) 16.5244781

7. What spreadsheet command should be in cell E1 of the above spreadsheet?

a.) (D1 - B1) / (C1 - A1)

b.) D1 / C1

c.) (D1 - C1) / (B1 - A1)

d.) (B1 - A1) / (D1 - C1)

e.) (C1 - A1) / (D1 - B1)

8. The formula for Newton's method to find a root of $f(x) = 0$ is:

a.) $x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$

b.) $x_{k+1} = x_k - \dfrac{f'(x_k)}{f(x_k)}$

c.) $x_{k+1} = x_k + \dfrac{f(x_k)}{f'(x_k)}$

d.) $x_{k+1} = f(x_k) - \dfrac{x_k}{f'(x_k)}$

e.) $x_{k+1} = \dfrac{x_k - f(x_k)}{f'(x_k)}$

9. If Newton's method is used to find the root of $x^2 - 5$ starting with $x_0 = 1$, which of the following will be the next two values?

a.) $x_1 = 3, \quad x_2 = 1/3$

b.) $x_1 = -1, \quad x_2 = -3$

c.) $x_1 = 3, \quad x_2 = 7/3$

d.) $x_1 = -1, \quad x_2 = 1$

e.) $x_1 = 3, \quad x_2 = -1$

10. Select the correct Maple command to evaluate

$$\int_0^\infty e^{-t} \cos(bt)\, dt$$

a.) `integrate:= exp(-t)*cos(b*t) , [0,infinity] ;`
b.) `int:= f(t), t, [0,infinity] ;`
c.) `integrate:= exp(-t)*cos(b*t) , t, 0..infinity ;`
d.) `int( exp(-t)*cos(b*t) , t= 0..infinity ) ;`
e.) `int( exp^(-t)*cos(b*t) , t= 0..infinity ) ;`

11. Consider the following Maple commands.

```
ex1:= cos(b) + sin(a) + a^2*b ;
ex2:= op( 3, ex1 ) ;
```

Given that `nops( ex1 )` is 3, which of the following will be `ex2`?

a.) 3

b.) `cos(b) + sin(a)`

c.) `sin(a)`

d.) `a^2`

e.) `a^2*b`

12. The trapezoid rule for approximating the integral

$$\int_A^B f(x)\ dx$$

has error equal to

$$-\frac{1}{12} f''(?)(B-A)h^2$$

where ? is some value in $[A:B]$. Consider estimating the integral

$$\int_{-1}^{2} F(x)\ dx$$

by the trapezoid rule. Assume that it is known that the second derivative of $F(\cdot)$ satisfies

$$|F''(x)| \leq 4$$

for x in the interval $[-1:2]$. Of the listed values, which is the *best* estimate of the number of intervals N required to compute the integral to an accuracy of 10^{-4} by the trapezoid rule?

a.) $N \geq 1000$ b.) $N \geq 500$ c.) $N \geq 300$

d.) $N \geq 100$ e.) $N \geq 50$

13. The formula for approximating the integral

$$\int_A^B g(t)\ dt$$

by the rectangle rule using N intervals of size h, and interval endpoints $t_j = A + jh$, is

a.) $h\sum_{j=0}^{N} g(t_j)$ b.) $h\sum_{j=1}^{N} g(t_{j-1})$ c.) $h\sum_{j=0}^{N} g(t_{j+1/2})$

d.) $h\sum_{j=0}^{N} g(t_{j-1/2})$ e.) $h\sum_{j=1}^{N} g(t_{j-1/2})$