An Introduction to Maple

CS 310 — Fall 1999

This handout provides an introductory "walk-through" for Maple. Maple is a computer algebra system that uses symbols to manipulate information.

To start Maple, go to
`Start -> CAE Applications -> Engineering -> M-S -> Maple 5.1`.

1 Getting Started

To get started, we'll enter a few formulas into Maple. Don't forget the ";" at the end of each of the following Maple commands. After you enter each statement, Maple will respond with what it thinks you entered. Before going to the next statement, be sure that each command was understood properly by Maple.

Maple Command	Comments
`f := (x^2+1)^4;`	Now `f` is $(x^2+1)^4$.
`eval(f,x=2);`	Evaluates `f` at $x=2$.
`expand(f);`	Maple expands the expression for `f`.
`g := (x-2*y)^3;`	Now `g` is $(x-2y)^3$.
`expand(g);`	Another expansion.
`h := subs(y=z^2-1, g);`	This sets `h` to what you get when you substitute $y=z^2-1$ into `g`.
`subs(x=2, z=1, h);`	This time we substitute numbers into `h`. The answer you get should be 8. See if you can find the value of `h` at $x=1, z=2$.
`k:= x -> x^2;`	This defines `k` as a function of `x`, i.e., $k(x)=x^2$. Functions of several variables can also be defined this way.
`k(3);`	Prints 9.

The above introduced two Maple commands, `expand` and `subs`. The "opposite" of `expand` is the `factor` command, which is used in the next set of examples.

Maple Command	Comments
`f := x^3 - 1;`	Now `f` is $x^3 - 1$.
`f := factor(f);`	Maple factors the expression for `f`, and reassigns the result back into `f`.
`expand(f);`	You should get back $x^3 - 1$ again.
`f;`	This will print the value of `f`. Note that it is still the factored version.
`g := x^4 + 1;`	Now we'll factor this expression.
`g := factor(g);`	This has no effect — we can't factor `g` over the integers.
`g := factor(g, sqrt(2));`	But we *can* factor `g` if we let the factors use $\sqrt{2}$ as a coefficient.
`g := subs(x=sqrt(y), g);`	Next we substitute $x = \sqrt{y}$ into `g`.
`expand(g);`	Expanding `g` results in $y^2 + 1$.

2 Integration

In this section we'll give some examples of using Maple to evaluate integrals. In the first example, we'll integrate an expression symbolically, and use the `simplify` command.

Maple Command	Comments
`i1 := int( 1/(1 + x^4), x);`	Integrate $\frac{1}{1+x^4}$ symbolically.

The next example does the same integration in a slightly different way.

`f := 1 + x^4;`	Set `f` to be the denominator of the integrand.
`i1 := int(1/f, x);`	Now integrate. The result is the same as above.

Maple can also simplify expressions:

`f:=sin(2*x)/sin(x);`	Set $f(x) = \frac{\sin(2x)}{\sin(x)}$.
`simplify(%);`	Shows the result: $2\cos(x)$.

In the above example, we used the abbreviation `%` — which means "the last result."

In the next example, we'll integrate another expression, but we'll do a definite integral.

Maple Command	Comments
`f := 1+x^4;`	Set `f` to be the denominator of the integrand.
`i1 := int(1/f, x=0..1);`	This integrates the expression from 0 to 1.
`simplify(i1);`	The expression is already simplified.
`evalf(%);`	This gives the numerical answer, 0.866...

As another example of definite integration, we will evaluate

$$\int_0^{\pi} \frac{1}{2+\sin(\theta)}\, d\theta$$

`int(1/(2+sin(t)), t=0..Pi);`	The answer is $2\sqrt{3}\pi/9$. You must use a capital `P` in `Pi`
`evalf(%);`	The answer is 1.209...

In the above example, we used the abbreviation `%` again, so the `evalf` command evaluated the results of the `int` command. Notice that by using `%` we didn't have to assign the results of the `int` command to a new variable. The ∞ symbol is written as `infinity` in Maple.

☞ Evaluate the integral

$$\int_0^{\infty} \frac{x^2-1}{(x^3+1)(x+2)}\, dx$$

(The correct answer, to 5 decimal places, is 0.12432)

3 Plotting

In this section we'll give some examples of using Maple to make a graph of a function. After the graphs are created, you can close them with the "Exit" option under the "File" menu.

Maple Command	Comments
`f := 4*x^4 - 2*x^2;`	`f` is the function we want to graph.
`plot(f, x=-1..1);`	Now Maple graphs `f` in the interval -1 to 1.

You can enlarge the graph by clicking on it. The graph will then be “framed” with black boxes at the corners. Click these corner boxes and drag them around to resize the graph. The next few commands give more practice graphing functions. The second command plots the function in the interval $(-2, 4)$. The third command plots the function again, specifying a vertical range for the plot.

```
f := ln(x^2 + 1) - exp(.4*x)*cos(Pi*x);

plot(f, x=-2..4);

plot(f, x=-2..4, y=-10..10);
```

☞ Make a plot of the following equation:

$$f(x) = 8x^4 - 8x^2 + 1 - e^x.$$

See if you can adjust the plot parameters to find all five roots.

4 Solving Equations

In this section, we'll use Maple's `solve` command to solve equations. The first example solves a quadratic equation.

Maple Command	**Comments**
`f := x^2-3*x+1;`	`f` is a quadratic function we will set equal to zero.
`factor(f);`	`f` can't be factored using integers.
`sol := solve(f=0, x);`	Now solve the equation $x^2 - 3x + 1 = 0$ for x. Notice that you must solve the *equation* instead of the *function*. The result is a list of the solution values.
`s1 := sol[1];`	This sets `s1` to the first root (the first element of `sol`, the solution list).
`s2 := sol[2];`	This sets `s2` to the second root (the second element of `sol`).
`max(sol);`	This finds the larger of the two roots.

Now we'll use the `solve` command to find the minimum value of the function

$$y = (x - 1)^4 - (x + 2)^2.$$

Remember that to find the extreme values of a function we need to take the derivative, then set that equal to zero.

`y := (x-1)^4 - (x+2)^2;`	Input the function y.
`plot(y, x=-1..4);`	From the plot, we see that the function has a minimum between 2 and 3.
`d := diff(y, x);`	Differentiate y with respect to x.
`sol := solve(d=0, x);`	Set the derivative equal to zero, and then solve for x.

The answer is quite messy. (Be thankful you didn't have to solve the equation by hand!) There are 3 roots. One is real, and the other two are complex (identified by the letter I).

We will only look at the one real root. This root is the "minimizer," which means that the function will attain its minimum value at this value of x.

`s1 := sol[1];`	Sometimes the real root won't be the first one in the list — choose whichever root (i.e., list element) doesn't have an I in it.
`simplify(s1);`	Simplify the result.
`xmin := evalf(%);`	Get a numerical value for the minimizer.
`val := subs(x=xmin, y);`	Determine the minimum — the function value at the minimizer.
`plot(y, x=-1..4);`	Make the plot again. (Rather than typing in the command again, you can "cut & paste" from above.) Can you verify that the values of `xmin` and `val` are correct from the plot?

The `solve` command uses analytical techniques to find the roots of equations. This is different from the methods you've used so far (such as Newton's method, the bisection method, etc.), which have all been numerical. If you want to use a numerical solution in Maple, you need to use the `fsolve` command.

The next example shows you how to solve an equation numerically using `fsolve`.

`f := x^2 - cos(x);`	This is the function $f(x)$.
`eq := f=0;`	This is the equation $f(x) = 0$.
`solve(eq);`	Maple fails to solve the equation analytically. In fact, the equation cannot be solved analytically — we must use numerical methods.
`fsolve(eq, x=0..Pi);`	With `fsolve`, you need to specify an interval to look for a solution in, just like with the NAG routine c05adf/c and the bisection method. Here, we have used the interval $(0, \pi)$. You'll get .824...
`plot(f, x=0..Pi);`	From the graph you can verify that a root was found correctly.

☞ Find the maximum value of the expression $\sin(\theta) + \cos(2\theta)$ in the interval $(0..\pi)$. The answer is 9/8. Make a plot of the function.

☞ Find where the two curves $x + (y - 1)^3 = 0$ and $y = -2x$ intersect. One answer is $(x, y) = (-1, 2)$, and another involves complex numbers.

5 Getting Help

Maple has an excellent help facility. To get help on a particular command, you can type "?*command*". For example, to get help on the `fsolve` command, you would type: `?fsolve`. Another way to get help isto highlight something in the worksheet and press `F1`.

6 Printing it out

The easiest way is to print is to click on the printer icon at the top of the screen. Or, you could select "Print" from the "File" menu. A new window will appear. From the window, you can either output to a postscript file, or directly print your Maple session. If you want to print to a file, type `f:\`*filename* in the box that comes up.

The print command will print your *entire* Maple session — everything you've done since you first started Maple. You can delete portions of your session by using the mouse to highlight the portion you don't want, and then selecting "Cut" from the "Edit" Menu.

When you print out your solutions for the homework problems, try to print out only those Maple commands that were necessary to solve the problem. You can delete all of your blunders and commands related to solving other problems.