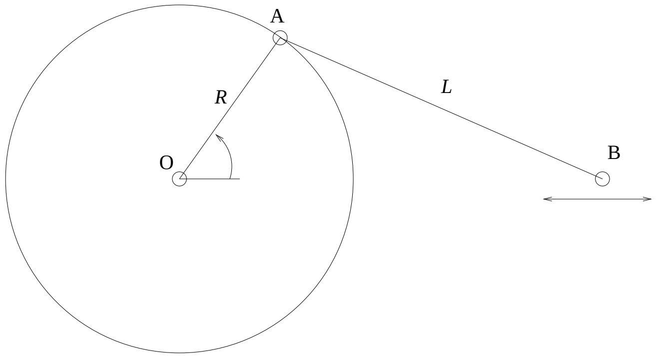
CS310 **Assignment 2** Fall 1999

Prof. Blanchard Due date: Oct. 5, 1999
Prof. Strikwerda Due date: Oct. 6, 1999

1. (30 points)

This problem concerns the mechanism shown in the figure. The wheel turns at a constant rate. The end marked B is constrained to move horizontally. The wheel has a radius of R and the arm A to B has length L. The wheel rotates with a constant angular rate of ω. Use a coordinate system centered at point O, the center of the wheel.



a. Find a set of equations relating the location of point B with the rotation of the wheel.
b. Find an expression for the speed of the point B as a function of t.
c. For the case with $R = 0.23$ m, $L = 1.34$ m, and $\omega = 5$ rev/sec, find the speed of the point B when point A is at the top of the wheel.

2. (50 points)

In this problem we seek the natural frequencies of the boom of a satellite. Satellites are launched in a compact configuration, but upon deployment they often have booms that are extended in order to serve some desired function. These booms must be designed such that they can withstand the loads they will experience during any planned or unplanned satellite motion. Since they are relatively long and must be lightweight (to minimize launch loads), they are subject to vibrations and must be designed to withstand prescribed vibrational or impulsive loads. A critical element of the design process is the understanding of the free vibration frequencies and mode shapes of the boom.

Free vibrations, also called natural vibrations, are oscillations experienced without external loading on the structure. They can be thought of as vibrations resulting from some initial loading which is then removed. Each vibration mode (or shape) has an associated vibration frequency. Structures such as satellite booms have an infinite number of mode shapes, each with an increasingly complexity. We generally must only concern ourselves with the simpler mode shapes, corresponding to the lower frequencies of vibration. Typical satellite booms can be modeled as beams. The equation of motion for such a beam is:

$$EI\frac{d^4y}{dx^4} - \rho\omega^2 y = 0$$

where E is the elastic modulus of the material, I is the moment of inertia of the cross section of the beam, ρ is the mass per unit length of the beam, ω is the natural frequency, x is the distance from the end of the beam, and y is the displacement of the beam. For a typical boom, the boundary conditions, assuming one end is fixed and the other free, are:

$$y(0) = 0, \quad \frac{dy}{dx}(0) = 0, \quad \frac{d^2y}{dx^2}(L) = 0, \quad \frac{d^3y}{dx^3}(L) = 0.$$

The general solution to the above differential equation is:

$$y(x) = A\cosh\beta x + B\cos\beta x + C\sinh\beta x + D\sin\beta x$$

where

$$\beta^4 = \frac{\rho\omega^2}{EI} .$$

We would like to predict the three smallest natural frequencies for a satellite boom with the following properties:

Bending stiffness = EI = 29,000 N m^3
Mass (per unit length)= ρ = 0.49 kg/m
Length = L =6.1 m

To find these frequencies, you must follow these steps using Maple:

a. Apply the four boundary conditions above to the general solution for the beam shape to derive four equations involving five unknowns. The five unknowns are β and the constants A, B, C, and D.
b. What you are to do is eliminate A, B, C, and D from the set of equations. What remains will be a transcendental equation which contains only β and L. One way to do this is to use the first three equations to eliminate A, B, and C from the fourth equation. At this point D will also drop out leaving the equation for β.
c. Solve this equation for the three smallest values of β.
d. From these values and the properties above, solve for the three smallest natural frequencies of the boom.

If you are unfamiliar with the functions $\cosh(x)$ and $\sinh(x)$, you can use Maple to plot them. Check the derivatives of these functions. Evaluate $\cosh(x)^2 - \sinh(x)^2$. Note that

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad \text{and} \quad \sinh(x) = \frac{e^x - e^{-x}}{2} .$$

You may work in pairs on this assignment, if you wish. Each pair will turn in one copy of your homework. Both students will receive the same grade.

Check the course web page for instructions on handing in the assignment.