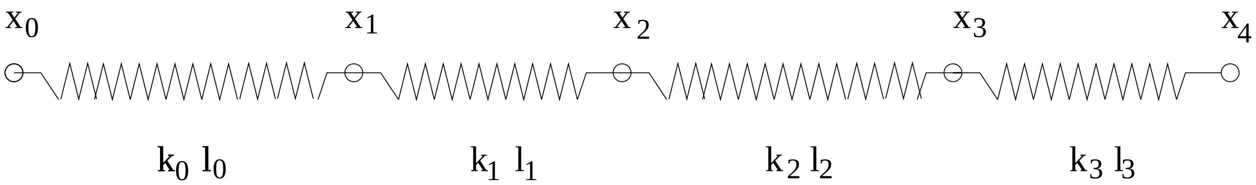
CS310 **Assignment 4** Fall 1999

Prof. Blanchard Due date: Nov. 9, 1999
Prof. Strikwerda Due date: Nov. 8, 1999

This assignment concerns using Matlab to solve linear systems of equations.

1. (15 points) In this problem you will solve for the position of several springs stretched between two points as shown in the figure.



Spring Diagram

The j-th spring has constant k_j and natural length ℓ_j as shown in the table.

j	0	1	2	3
k_j	1.5	1.0	1.5	2.0
l_j	0.3	0.1	0.2	0.2

Spring Data

The total length that the springs will be stretched is 1. (We will not need to specify units for this example.)

The equation determining the position of the masses is the balance of forces between adjacent springs. The force exerted by a spring is the spring constant times the distance stretched beyond the natural length.

$$k_{j-1}(x_j - x_{j-1} - \ell_{j-1}) = k_j(x_{j+1} - x_j - \ell_j)$$

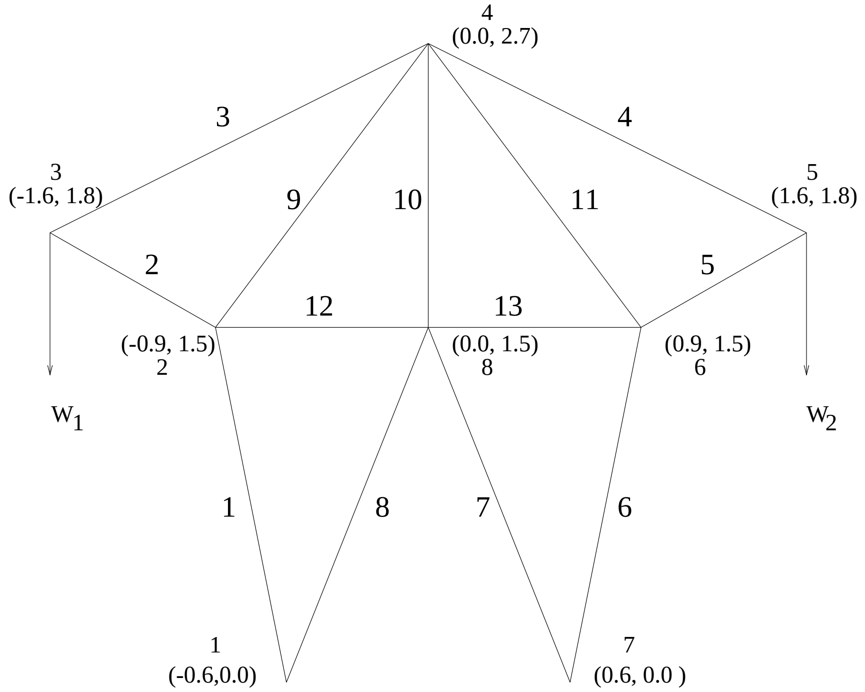
Rearranging this to get it in matrix form, the equations are:

$$-k_{j-1}x_{j-1} + (k_{j-1} + k_j)x_j - k_j x_{j+1} = k_{j-1}\ell_{j-1} - k_j\ell_j$$

for $j = 1, 2$, and 3.

Use Matlab to set up the matrix equations to determine the positions x_j and solve the linear system. Using the Matlab operator `diag` can make the construction of the matrix easier.

2. (35 points) Consider the structure in the figure. You are to determine the forces on the different members given that $W_1 = 10$ and $W_2 = 15$. The coordinates of the joints are given in the figure. The joints are numbered above or below the coordinates and the members are designated by numbers next to the members.



Structure Diagram

The equation at each joint is the balance of the forces. For example, at joint 4, with coordinates $(0.0, 2.7)$ at the top of the structure, the force equations are:

$$F_3 \cos\theta_{3,4} + F_4 \cos\theta_{5,4} + F_9 \cos\theta_{2,4} + F_{10} \cos\theta_{8,4} + F_{11} \cos\theta_{6,4} = 0$$
$$F_3 \sin\theta_{3,4} + F_4 \sin\theta_{5,4} + F_9 \sin\theta_{2,4} + F_{10} \sin\theta_{8,4} + F_{11} \sin\theta_{6,4} = 0$$

At joints 3 and 5, the y component of force must balance with the applied forces W_1 and W_2, respectively.

The quantities $\cos\theta_{i,j}$ and $\sin\theta_{i,j}$ are the cosine and sine of the angles between joints i and j measured from the horizontal at joint j. They are given by

$$\cos\theta_{i,j} = (x(i) - x(j)) / \text{ dist}_{i,j}$$
$$\sin\theta_{i,j} = (y(i) - y(j)) / \text{ dist}_{i,j}$$

where $\text{dist}_{i,j}$ is the distance between the joints, i.e.,

$$\text{dist}_{i,j} = ((x(i) - x(j))^2 + (y(i) - y(j))^2)^{1/2}$$

You are to set up and solve the matrix equations using Matlab and then solve the linear system for the forces on the members

The Equations

There are two equations per joint, giving 16 equations. There are only 13 forces for which to solve. Three equations are to be dropped. The dropping of these equations is related to the structure being held in place. Drop the x and y force equations at joint 1 and the y force equation at joint 7. This gives 13 equations for 13 forces. The system is nonsingular, that is, it has a solution.

Interpreting The Results

If the force on a member is positive, then it is a compression force, if it is negative, then it is an extension force.

WRITING YOUR MATLAB PROGRAM

Set Up Some Arrays

To set up the matrix in a Matlab program, you should create arrays giving the coordinates of the points and an array giving the connectivity of the structure. You might have a coordinate array xy so that $xy(j,1)$ is the x coordinate and $xy(j,2)$ is the y coordinate of joint j. Alternatively, you could have arrays x and y for the coordinate components.

Set up the connectivity array so that if member m connects joints j_1 and j_2, then $C(m,1) = j_1$ and $C(m,2) = j_2$.

The Columns And Rows Of The Matrix

Note that each column of the matrix corresponds to a member and each row corresponds to an equation. A member contributes to only the equations that give the force balance at the joints at the ends of the member. Thus each column will have at most four non-zero entries.

Assign Equations To Rows Of The Matrix

To fill in the column corresponding to a member, you will have to know to which equations that member contributes and what are those equation numbers. To do this you must construct a rule that gives the equation number for the x and y balance equations at each joint. For example, the x balance equation at joint 5 might be equation number 8, and the y equation at joint 8 might be equation 13.

For most of the equations you should be able to develop a simple mathematical rule that associates equation numbers with joint numbers. Different students may have different rules for numbering the equations.

How To Construct The Matrix

Using a Matlab 'for' loop over the members, you can set up the matrix. Each member contributes to the equations at only 2 joints. The connectivity array tells you to which joints the member is connected. Given a joint, you should use the rule for determining to which equations the x and y force balance at the joint correspond. Put the value of $\cos\theta_{i,j}$ or $\sin\theta_{i,j}$ in the appropriate row of the matrix. What are i and j?

Helps With Programming

Include comments in your Matlab program so that you remember what you are doing. Comments are lines that begin with %.

As part of your comments include a list showing your equation numbering. That is, show which equations, i.e., joint and x or y, correspond to which rows.

Use good variable names and some conventions to determine your variable names. For example, you might use jnt, $jnt1$, etc. to refer to joints and use mem, $mem1$, etc. to refer to members.

What To Hand In

Hand in a listing of your Matlab program and the results of executing your program. You will have to use a *.m* file to store your program.

Print out the matrix as part of your program.

You can get a record of your Matlab run using the Matlab command 'diary'. Note that between executing 'diary' or 'diary on' and 'diary off', all of your Matlab output is saved on a file called `diary`. Each run of Matlab will append the output to `diary` so you will have to edit the file or delete it between Matlab runs.

A Help

If W_1 and W_2 are equal, then the forces on the structure are symmetric, i.e., the force on member 3 is the same as on member 4. If $W_1 = W_2 = 10$, then the forces on members 12 and 13 are 7.

Another Help

Note that $\cos\theta_{i,j} = -\cos\theta_{j,i}$ and similarly for $\sin\theta_{i,j}$. If you mess up the signs of the coefficients in the matrix, you will get incorrect answers.