CS310 **Assignment 5** Fall 1999

Prof. Blanchard Due date: Dec. 2, 1999

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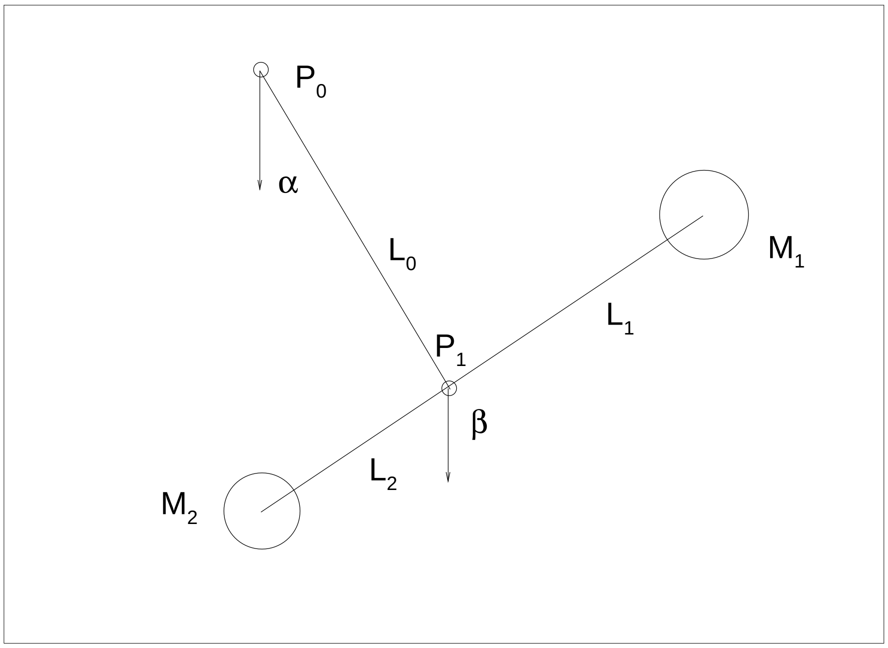
This problem has to do with a compound pendulum whose motion is chaotic for most starting points. The pendulum is displayed in Figure 1. It consists of two masses on the ends of a bar that is free to rotate on the end of a pendulum. The length of the pendulum is L_0, and the bar holding the masses has length $L_1 + L_2$. Mass M_1 is at a length L_1 from the point of contact with the pendulum. Similarly, mass M_2 is at a length L_2 from the point of contact with the pendulum.



The coordinates that describe the position of the pendulum system are the two angles α and β. The angle α is measured from the vertical down from the top pivot point P_0 to the bar with length L_0. The angle β is measured from the vertical down from the pivot point P_1 at the end of the bar L_0 to the portion of the bar with length L_1 and holding mass M_1.

The quantities that determine the system are the three lengths and the two masses. In addition we define the quantity

$$B = \frac{M_1 L_1 - M_2 L_2}{M_1 + M_2} .$$

The magnitude of B is a measure of how balanced the two masses are around the pivot point.

The motion is determined by the masses and lengths, and also the initial conditions for the motion. The equations are:

$$L_0\,\ddot{\alpha} + B\cos(\alpha-\beta)\,\ddot{\beta} = -B\sin(\alpha-\beta)\,\dot{\beta}^2 - G\sin\alpha$$

$$B\cos(\alpha-\beta)\,\ddot{\alpha} + \frac{M_1L_1^2 + M_2L_2^2}{L_0(M_1+M_2)}\,\ddot{\beta} = +B\sin(\alpha-\beta)\,\dot{\alpha}^2 - \frac{GB}{L_0}\sin\beta$$

We use the dot over the variable to denote the derivative with respect to time. G is the gravitational acceleration. Notice that the motion depends only on the relative masses, so we will restrict to the case with $M_1 + M_2 = 1$. To reduce the number of parameters we will only consider cases with $L_0 = 1$ and with $L_1 + L_2 = 1$.

For most initial conditions the motion of the masses is rather chaotic. That is, it is not a periodic motion nor does it have any nice pattern.

For special cases the solution of this system is periodic. One special case is if the masses balance around the pivot point. In this case $B = 0$, that is, $L_1M_1 = L_2M_2$. In this case the angle β remains constant and the pendulum moves as if the total mass $M_1 + M_2$ is located at the pivot point P_1.

Copy the files `chaotic.m` and `pendulum.m` from the course Web page. The program on `chaotic.m` solves these equations using the file `pendulum.m`. The code uses the Matlab library routine ode45. You should run this code for some cases. In this code the pendulum starts at rest with α and β given by the user.

There are two things you are asked to do with this code.

1. (50 points) Modify the equations to include a third pendulum of negligible mass that hangs from mass M_1. This is shown in Figure 2. The equation for the third angle γ is

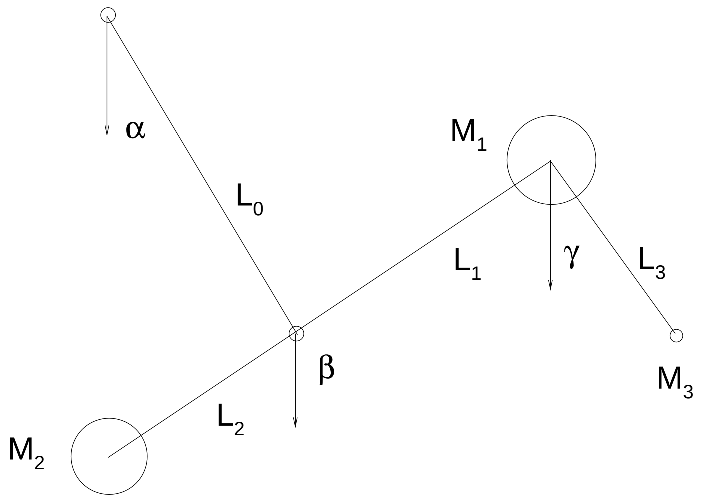
$$\begin{aligned} L_3\,\ddot{\gamma} = &- L_0\cos(\alpha-\gamma)\,\ddot{\alpha} - L_1\cos(\beta-\gamma)\,\ddot{\beta} \\ &+ L_0\sin(\alpha-\gamma)\,\dot{\alpha}^2 + L_1\sin(\beta-\gamma)\,\dot{\beta}^2 - G\sin\gamma \end{aligned}$$

Because the mass M_3 is negligible compared with M_1 and M_2, the motion of M_3 does not affect the motion of M_1 and M_2. We will restrict to the case with $L_3 = L_1$.

2. (30 points) Plot the motion of M_3 with points (using circles) along with the motion of M_1.

Here are several cases of possible interest.

1) A case that appears to be periodic, or very nearly so, is with $\alpha = \beta = \pi/2$ and $L_1 = 0.75$, $L_2 = 0.25$, $M_1 = 0.75$, $M_2 = 0.25$.



Two cases that appear to be chaotic are with $\alpha = \pi/2$, $\beta = \pi/4$ and

2) $L_1 = 0.5$, $L_2 = 0.5$, $M_1 = 0.7$, $M_2 = 0.3$.

and

3) $L_1 = 0.7$, $L_2 = 0.3$, $M_1 = 0.6$, $M_2 = 0.4$.

What To Hand In

Hand in a copy of your Matlab code with the changes you made clearly marked or highlighted.

Run the case with $\alpha = \pi/2$, $\beta = \pi/4$, $L_1 = 0.5$, $L_2 = 0.5$, $M_1 = 0.7$, $M_2 = 0.3$. and $\gamma = 0$. Hand in plots displaying the result.